

Interactions of PDMS with a surface.

Shirish M. Chitanvis, Group T-12

- Reinforcement of PDMS with filler particles leads to increased strength.

Q. What is the nature of the local reinforcement mechanism ?

A. Hooking between polymers adsorbed on the filler surface and the surrounding matrix.



The probability distribution for a single flexible chain is:

$$\begin{aligned} G_0(1, 2; n) &\equiv \langle 1, n | \left[\partial_n - \left(\frac{b^2}{6} \right) \nabla^2 \right]^{-1} | 2, 0 \rangle \\ &\sim \int_{\vec{R}_2}^{\vec{R}_1} \mathcal{D}\vec{R}(n') \exp - \left[\left(\frac{3}{2b^2} \right) \int_0^n dn' \left(\frac{\partial \vec{R}(n')}{\partial n'} \right)^2 \right] \\ &\sim \left(\frac{2\pi n b^2}{3} \right)^{-3/2} \exp - \left[\frac{3(\vec{R}_1 - \vec{R}_2)^2}{2n b^2} \right] \end{aligned} \quad (1)$$

- where $\partial_n \equiv \frac{\partial}{\partial n}$.
- This expression is obtained by considering only the entropy of a flexible chain.

Alternatively,

$$\begin{aligned} \langle 1, n | \left[\partial_n - \left(\frac{b^2}{6} \right) \nabla^2 \right]^{-1} | 2, 0 \rangle &\sim \\ \int \mathcal{D}^2 \psi \, \psi^*(\vec{R}_1, n) \psi(\vec{R}_2, 0) \exp -[\beta \mathcal{F}] \\ \beta \mathcal{F} &= \int dn' d^3 x \, \psi^*(\vec{x}, n') \left[\partial_{n'} - \left(\frac{b^2}{6} \right) \nabla^2 \right] \psi(\vec{x}, n') \quad (2) \end{aligned}$$

- Thus we have another way of thinking about a system of flexible polymers, in terms of $\psi(\vec{x}, n)$ and an energy functional $\beta \mathcal{F}$ which is isomorphic to one that describes diffusion.

- (\vec{x}, n) labels the location \vec{x} in physical space, of the n -th segment of a chain, and $|\psi(\vec{x}, n)|^2$ is the probability of finding a polymer segment at a given location in space.
- This is a density functional theory in the style of Kohn and Sham.
- Contains a description of *many* independent polymers. Hence more powerful than the single chain approach.

Density Functional formalism for *interacting* polymers

$$\mathcal{Z} = \int \mathcal{D}^2\psi \exp -[\beta\mathcal{F}] \text{ [Partition Function]}$$

$$\beta\mathcal{F} = \int dn d^3x \psi^*(\vec{x}, n) [\partial_n + H_0 + V] \psi(\vec{x}, n)$$

$$V = \frac{v}{2} |\psi(\vec{x}, n)|^2 - \mu$$

$$H_0 \equiv - \left(\frac{b^2}{6} \right) \nabla^2 \tag{3}$$

Extremizing the functional yields a Schrodinger-like equation (Hartree-Fock-like approximation):

$$\left[\frac{\partial}{\partial n} - \left(\frac{b^2}{6} \right) \nabla^2 - \mu_0 + v_{eff} |\psi(\vec{x}, n)|^2 \right] \psi(\vec{x}, n) = 0 \quad (4)$$

- $v_{eff} = v - w$, $1/2w$ is the probability per unit volume for cross-linking.

- Theory can be used to re-evaluate the basic physics of polymer melts, encapsulated in Flory's theorem:

At very low concentrations, fluctuations in the melt are high, but at high concentrations, they are so suppressed that the polymers begin to behave independently again.

- Can show that $R_g \sim N^\nu$, $\nu \approx 0.631$, for $c_0 \rightarrow 0$.
- Can show for $c_0 \rightarrow 0$, chains take on the appearance of a pearl necklace.
- Can show that the transition to entanglement is also a critical phenomenon, such that $R_g \sim (N^{-1} - 2\mu^{-1})^{-\nu}$.
- A tube model can be recovered for higher concentrations.

Surface Interactions

- Assume that adsorption occurs in a non-preferential manner, so that all segments of a polymer are equally likely to become attached to the surface. Take $\psi(\vec{r}, n) \equiv f(z)$:

$$\frac{d^2 f(z)}{dz^2} + f(z) - f^3(z) = 0 \quad (5)$$

where the length scale is $a = b/\sqrt{6c_0v_{eff}}$ and the field has been scaled by $1/\sqrt{c_0}$, c_0 being the monomer number density.

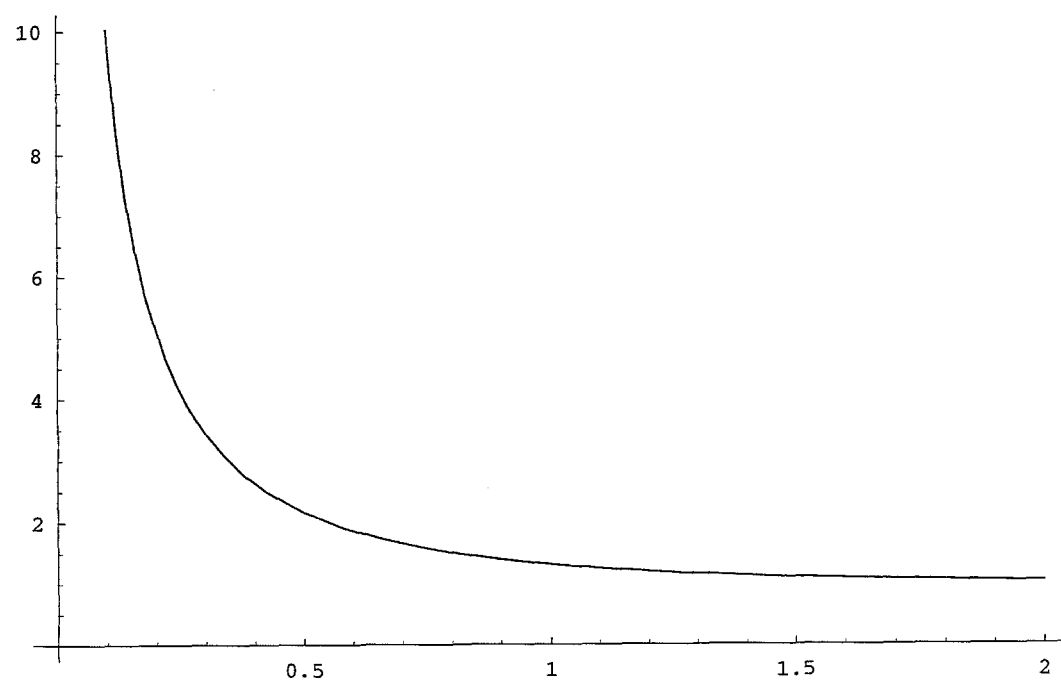
- Two solutions possible:

$$\begin{aligned} f_t(z) &= \coth \left(\frac{z - z_t}{\sqrt{2}} \right) [\mathbf{Total}] \\ f_m(z) &= \tanh \left(\frac{z - z_m}{\sqrt{2}} \right) [\mathbf{Matrix}] \end{aligned} \quad (6)$$

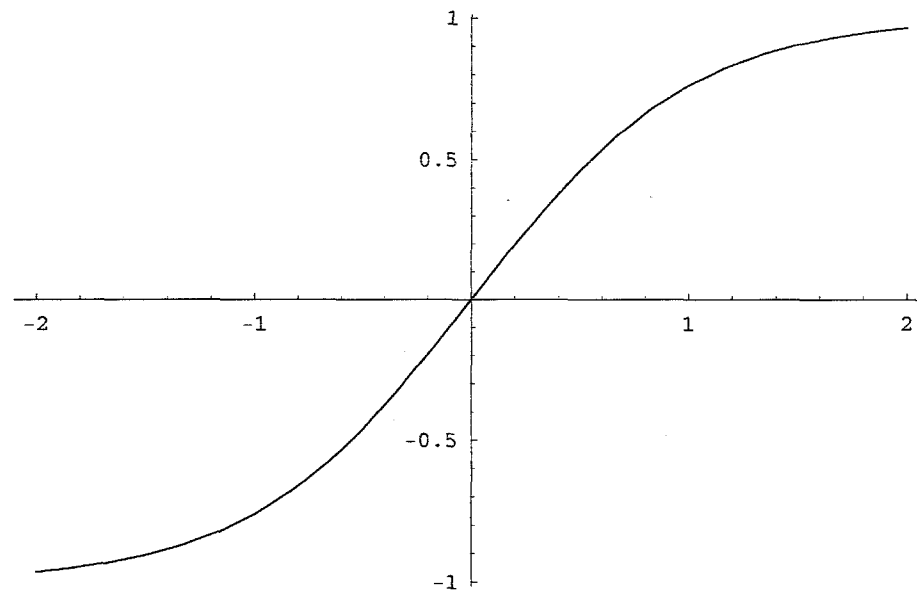
- Boundary condition:

$$- \left(\frac{d \ln f_t(z)}{dz} \right)_{z=0} = \lambda^{-1} \quad (7)$$

Coth function



Tanh function



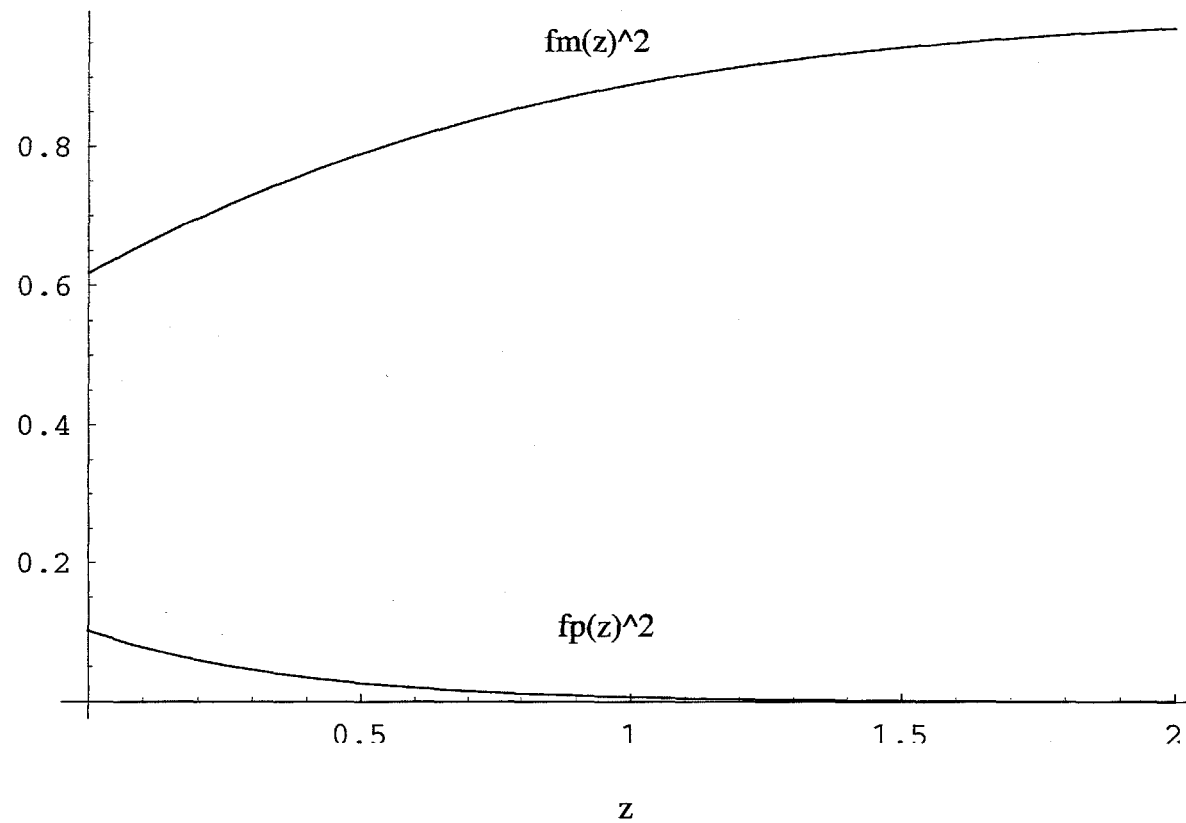
The profile of the pseudo-brush is given by the square of the following function:

$$f_p(z) = \coth\left(\frac{z - z_t(\lambda)}{\sqrt{2}}\right) - \tanh\left(\frac{z - z_m(\lambda)}{\sqrt{2}}\right) \quad (8)$$

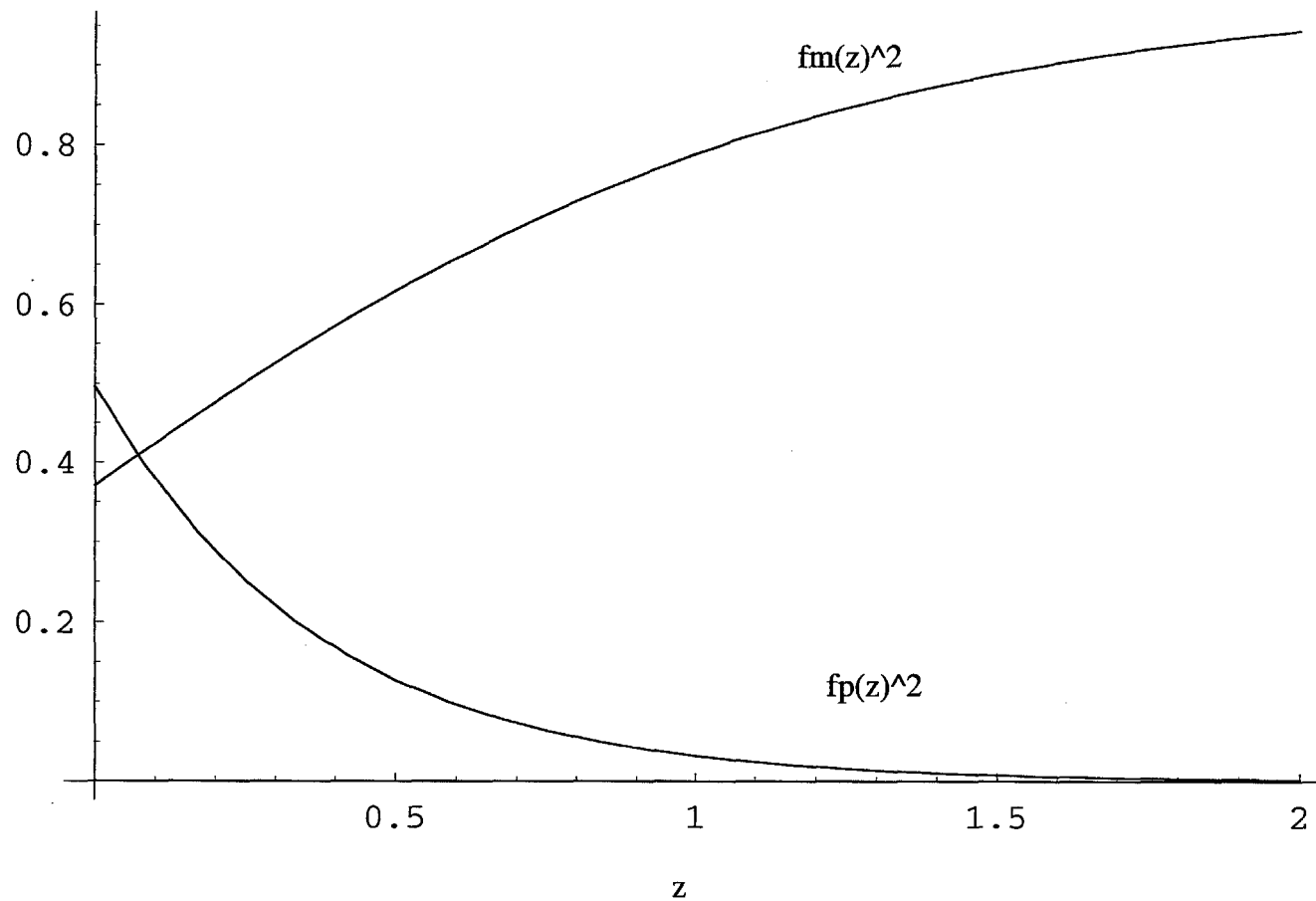
• Grafting density:

$$\frac{\sigma}{b^2} = c_0 \lambda a |f_p(z = 0)|^2 \quad (9)$$

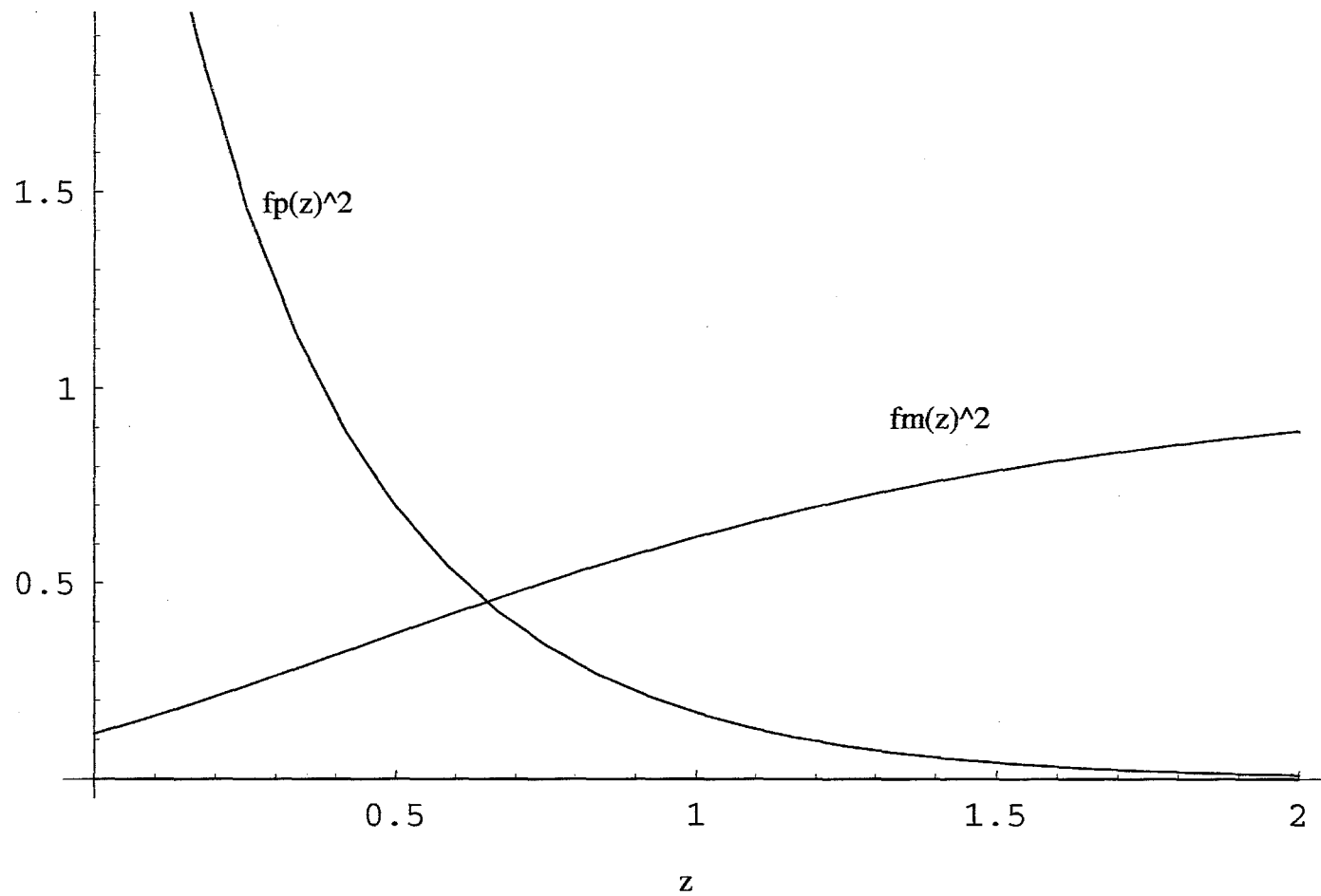
$\lambda = 1.5$ (weak adsorption)



$\lambda = 1.0$ (near optimal adsorption)



$\lambda = 0.5$ (stronger adsorption)



- A measure of interdigitation (overlap):

$$\begin{aligned}\mathcal{I}(\lambda) &= \int_0^\infty dz \, c_p(z, \lambda) [1 - c_p(z, \lambda)] \\ c_p(z, \lambda) &= \left[\frac{f_p(z, \lambda)}{f_p(z=0, \lambda)} \right]^2\end{aligned}\tag{10}$$

- $\lambda_{optimal} \sim 1$, implying $\sigma_{optimal} \sim 0.02$, for PDMS.

